Programme and Abstracts

The 41st Australasian Conference on Combinatorial Mathematics and Combinatorial Computing

Millennium Hotel, Rotorua, New Zealand
December 10–14, 2018

Combinatorial Mathematics Society of Australasia (CMSA)

Computing and Mathematical Sciences, The University of Waikato

All plenary talks will be in Mokoia. Contributed talks will be in Millennium One, Mokoia, or Millennium Five (Monday only: Millenium Four, Mokoia, Nikau Restaurant) depending on whether they are in the left, middle or right column of the timetable respectively.

Sunday 9 December

17:30-19:00 Registration and Welcome Party
Millennium Hotel
## Monday

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<td><strong>Mike Steel</strong> <em>Phylogenetic questions inspired by the theorems of Arrow and Dilworth</em> (Conder)</td>
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| 10:30  | Kristina Wicke *Combinatorial views on persistent characters* (Steel)  
Jamie Simpson *Palindromes in Starlike Trees* (Glen)  
Binzhou Xia *Stability of circulant graphs* (Conder) |
| 11:00  | Jonathon Klawitter *The agreement distance of phylogenetic networks* (Steel)  
Tim E. Wilson *An Axiomatic Unification of Graph Colouring* (Glen)  
Pongpat Sittitrai *Defective 2-colorings of planar graphs without 4-cycles and 5-cycles* (Conder) |
| 11:30  | Haya Aldosari *Spanning hypertrees in uniform hypergraphs* (Steel)  
Simone Linz *Tight Kernels in Phylogenetics* (Glen)  
Tomáš Vetrík *General Multiplicative Zagreb Indices of Trees* (Conder) |
| 12:00  | Lunch                           |
| 14:00  | **Ian Wanless** *Applications of matrix permanents in the study of Latin squares* (Cavenagh)  
Jane (Pu) Gao *The rank of random matrices over finite fields* (McLeod)  
Klaus Jansen *Empowering the Configuration-IP — New PTAS Results for Scheduling with Setups Times* (Petterson) |
| 15:00  | Afternoon Tea                   |
| 15:30  | Angus Southwell *Uniform generation of random Latin rectangles* (Royle)  
Jane (Pu) Gao *The rank of random matrices over finite fields* (McLeod)  
Klaus Jansen *Empowering the Configuration-IP — New PTAS Results for Scheduling with Setups Times* (Petterson) |
| 16:00  | Murray Tannock *Prolific patterns on combinatorial structures* (Royle)  
Mikhail Isaev *The sandwich conjecture of random regular graphs and more* (McLeod)  
Trent G. Marbach *Balanced Equi-n-squares* (Petterson) |
| 16:30  | Jinge Li *Uniquely Wilf Permutation Classes* (Royle)  
Catherine Greenhill *Approximately counting independent sets in graphs with bounded bipartite pathwidth* (McLeod) |
<p>| 17:00  | CMSA AGM (Mokoia)               |</p>
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<td>Jesse Lansdown</td>
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### Thursday

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<td>The Hamilton-Waterloo problem</td>
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<td>10:30</td>
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<td>Srinibas Swain</td>
<td>An Online Graph Atlas</td>
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<td>Peter Nelson</td>
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<td>Marston Conder</td>
<td>New methods for finding minimum genus embeddings of graphs</td>
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<td>Billy Child</td>
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<td>Sara Herke</td>
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<td>Dillon Mayhew</td>
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<td>William Pettersson</td>
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<td>Keisuke Shiromoto</td>
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<td>Characterizations of hyperplanes related to quadrics in projective space</td>
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<td>A Stallings’ type theorem for quasi-transitive graphs</td>
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<td>Santiago Barrera Acevedo</td>
<td>Coyclic Hadamard matrices of order 4p</td>
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<td>Mark C. Wilson</td>
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<td>Gordon Royle</td>
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<td>Paul Leopardi</td>
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<td>Duy Ho</td>
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**Conference Dinner**

Venue: The Blue Baths

Arrival: 18:30-18:45. Dinner served: 19:00
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Applications of algebraic combinatorics to problems in finite geometry

John Bamberg*

*University of Western Australia

When an extremal configuration comes to mind, one may think of a cage (in graph theory), a Steiner system (in design theory), a perfect code (in coding theory), or an ovoid of a finite polar space (in finite geometry). In many cases, there is an algebraic way to express the extremal phenomenon which introduces, for example, eigenvalue bounds that can provide insight into the existence of such configurations. Delsarte defined a combinatorial object known as a T-design (which is more general than a t-design), as a particularly regular subset with respect to a given association scheme. This allowed extremal configurations from seemingly disparate areas of combinatorics to be studied with unified techniques. In this talk, the speaker will give an overview of the connection between algebraic combinatorics (e.g., association schemes) and problems in finite geometry.

(Friday 9:00)

The Hamilton-Waterloo problem

Peter Danziger*

*Ryerson University, Toronto

Given a graph $G$, a 2-factor is a spanning subgraph of $G$ where every vertex has degree 2, and is hence a collection of cycles. A 2-factorization of $G$ is a set of 2-factors that between them partition the edges of $G$. Given a graph $G$, two 2-factors $F_1$ and $F_2$ and non-negative integers $\alpha$ and $\beta$, the Hamilton-Waterloo problem, HWP($G; F_1, F_2; \alpha, \beta$) asks for a partition of the edges of $G$ into $\alpha$ factors isomorphic to $F_1$ and $\beta$ factors isomorphic to $F_2$.

In this talk we concentrate on the uniform case, where $F_1$ is a collection of $n$-cycles ($C_n$'s) and $F_2$ is a collection of $m$-cycles ($C_m$'s). Without loss of generality we may assume that $n \geq m \geq 3$. Specifically, given non-negative integers $v, m, n, \alpha, \beta$, the Hamilton-Waterloo problem, HWP($G; n, m; \alpha, \beta$), asks for a factorization of the complete graph, $G$ into $\alpha$ $C_n$-factors and $\beta$ $C_m$-factors. Classically, $G$ is $K_v$ (or $K_v$ minus a 1-factor when $v$ is even), in which case $m \mid v$, $n \mid v$ and $\alpha + \beta = (v - 1)/2$ are necessary conditions. However, other options for $G$, such as a 'blowup' of a complete graph or a cycle are also relevant in obtaining solutions to the original problem.

Recently there have been significant advances in the Hamilton-Waterloo problem. This talk will discuss these advances and relate some of the methods that have been used.

(Thursday 9:00)
Quantum problems for Graph theorists

ChrisGodsil∗
∗University of Waterloo

Quantum physics has a number of surprising features, one of which is that work on quantum computation has lead to a number of interesting problems in graph theory. My talk will provide an introduction to some of these problems.

I will focus on two classes of problems. The first arise from quantum analogs of random walks on graphs, and lead to a number of questions related to spectral graph theory. The second class concerns colouring problems. These may be questions about the usual colourings, as well as questions involving “quantum” analogs of classical colourings.

(Tuesday 9:00)

Polytopes, graphs and groups

Isabel Hubard∗ and Elías Mochán
∗Instituto de Matemáticas, Universidad Nacional Autónoma de México

Abstract polytopes generalise the classical notion of geometric polytopes to more general structures. Examples of abstract polytopes include convex and starry polytopes, as well as (some) maps on surfaces. The combinatorial structure of \( n \)-polytopes is completely determined by an edge-coloured \( n \)-valent graph with chromatic index \( n \), often called the flag graph.

Traditionally, the main focus of the study of maps and polytopes has been their symmetries. Of particular interest are the maps and polytopes that are regular (having the maximum degree of symmetry) and chiral (having the maximum degree of symmetry by rotation). In this talk we will be interested in highly symmetric polytopes that are not necessarily regular or chiral.

Symmetry type graphs (that is, the quotient of its flag graph under the action of the automorphism group) are a great tool to study highly symmetric polytopes. In particular one can obtain a generating set for the automorphism group of a polytope, given its symmetry type graph.

Not all edge-coloured \( n \)-valent graphs with chromatic index \( n \) are flag graphs of polytopes. We give conditions for such a graph to be the flag graph of a polytope, and then use these conditions to give conditions on a group, together with a set of generators, to be the automorphism group of a polytope. These last conditions generalise the so-called intersection condition of regular and chiral polytopes.

This talk will include some work with Gabe Cunningham, María del Río and Micael Toledo, with Jorge Garza and with Elías Mochán.

(Thursday 14:00)
Ramsey equivalence in many colours

Dennis Clemens, Anita Liebenau* and Damian Reding.

*University of New South Wales, Sydney

Two graphs $H$ and $F$ are called $q$-Ramsey equivalent if any graph $G$ is $q$-Ramsey for $H$ if and only if it is $q$-Ramsey for $F$ (where $G$ is $q$-Ramsey for $H$, say, if every colouring of the edges of $G$ with $q$ colours contains a monochromatic copy of $H$). Fox et al. proved that the only connected graph which is 2-Ramsey equivalent to the complete graph $K_n$ is $K_n$ itself. It is open whether there exist two connected non-isomorphic graphs that are 2-Ramsey equivalent. This question may be easier to answer for $q > 2$ colours. If $H$ and $F$ are (not) 2-equivalent then this does not imply a priori that they are (not) $q$-equivalent for $q > 2$. We address questions on $q$-Ramsey equivalence for $q > 2$. Using similar methods we prove that, for every 3-connected graph $H$, every graph $G$ which is minimal 2-Ramsey for $H$ is contained as an induced subgraph in a graph $G'$ which is minimal $q$-Ramsey for $H$, for all $q > 2$.

(Tuesday 14:00)

Phylogenetic questions inspired by the theorems of Arrow and Dilworth

Mike Steel*

*University of Canterbury

Biologists frequently need to reconcile conflicting estimates of the evolutionary relationships between species by taking a ‘consensus’ of a set of phylogenetic trees. This is because different data and/or different methods can produce different trees. If we think of each tree as providing a ‘vote’ for the unknown true phylogeny, then we can view consensus methods as a type of voting procedure. Kenneth Arrow’s celebrated ‘impossibility theorem’ (1950) shows that no voting procedure can simultaneously satisfy seemingly innocent and desirable properties. We consider a similar axiomatic approach to consensus and ask what properties can be jointly achieved. In the second part of the talk, we consider phylogenetic networks (which are more general than trees as they allow for reticulate evolution). The question ‘when is a phylogenetic network merely a tree with additional links between its edges?’ is relevant to biology and interesting mathematically. It has recently been shown that such ‘tree-based’ networks can be efficiently characterised. We describe some further results related to Dilworth’s theorem for posets (1950), and matching theory in bipartite graphs. In this way, one can obtain fast algorithms for determining when a network is tree-based and, if not, to calculate how ‘close’ to tree-based it is.

(Monday 9:10)
Applications of matrix permanents in the study of Latin squares

Ian Wanless*

*Monash University

The permanent of an $n \times n$ matrix $A = [a_{i,j}]$ is defined by

$$\text{per } A = \sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i,\sigma(i)}$$

where the sum is over all permutations in the symmetric group $S_n$. The permanent is useful in many counting problems. In this survey talk, I will concentrate mainly on its applications to the study of Latin squares, although some applications in graph theory will also be mentioned.

A latin square of order $n$ is an $n \times n$ matrix in which $n$ distinct symbols are arranged so that each symbol occurs once in each row and column. Examples include Cayley tables of finite groups and completed sudoku puzzles. A transversal in a latin square is a selection of $n$ distinct entries in which each row, column and symbol is represented exactly once. There are several infamous open problems dealing with transversals in Latin squares. I’ll solve none of them! Instead I’ll survey a variety of ways in which permanents arise when asking basic questions about Latin squares, including enumerations and existence questions for transversals, to name just two.

(Monday 14:00)
Abstracts of contributed talks

Spanning hypertrees in uniform hypergraphs

Haya Aldosari* and Catherine Greenhill
*University of New South Wales, Sydney

An \( r \)-uniform hypergraph \( H = (V,E) \) is a generalisation of a graph where its edges are \( r \)-element subsets of \( V \). A spanning hypertree \( T = (V(T),E(T)) \) in \( H \) is a connected and acyclic hypergraph with \( E(T) \subseteq E \) and \( V(T) = V \). I will talk about the expected number of spanning \( r \)-uniform hypertrees in random \( r \)-uniform hypergraphs. This is the analogue of a formula for the average number of spanning trees in the graph case obtained by Greenhill et al. in 2017.

(Monday 11:30)

Lines in two-coloured space

Andrii Arman*
*Monash University

A general problem in Euclidean Ramsey theory is for two geometric configurations of points to determine if any colouring of a Euclidean space in two colours (red and blue) contains a congruent monochromatic copy of one of the configurations. Colourings of a space in red and blue in which there are no two red points distance one apart are called almost blue.

In this talk I will give a short survey on almost blue colourings and show that any almost blue colouring of three dimensional Euclidean space contains six blue collinear points separated by unit distances. This talk is based on a joint work with Sergei Tsaturian.

(Friday 11:00)
Cocyclic Hadamard matrices of order $4p$

Santiago Barrera Acevedo*, Padraig Ó Catháin and Heiko Dietrich

*Monash University

Cocyclic Hadamard matrices were introduced by de Launey and Horadam as a class of Hadamard matrices with interesting algebraic properties. Catháin and Röder described a classification algorithm for cocyclic Hadamard matrices of order $4n$ based on relative difference sets in groups of order $8n$; this led to the classification of cocyclic Hadamard matrices of order at most 36. Based on work of de Launey and Flannery, we investigated in detail the structure of cocyclic Hadamard matrices of order $4p$, with $p$ prime. This led to a classification algorithm and the determination of cocyclic Hadamard matrices of orders 44 and 52 up to equivalence.

(Thursday 16:00)

Star decompositions of the complete multigraph

Rosalind Cameron* and Daniel Horsley

*Memorial University of Newfoundland

In 1979 Tarsi showed that an edge decomposition of a complete multigraph into stars of size $k$ exists whenever the obvious necessary conditions hold. In 1996 Lin and Shyu gave necessary and sufficient conditions for the existence of an edge decomposition of a (simple) complete graph into stars of sizes $m_1, \ldots, m_t$.

We show that the general problem of when a complete multigraph admits a decomposition into stars of sizes $m_1, \ldots, m_t$ is NP-complete, but that it becomes tractable if we place a strong enough upper bound on $\max(m_1, \ldots, m_t)$. We determine the upper bound at which this transition occurs and discuss some of the complications that arise in this generalisation.

(Wednesday 09:30)

Defining sets of full designs and full Latin squares

Nicholas Cavenagh*

*University of Waikato

The full $n$-Latin square is the $n \times n$ array with symbols $1, 2, \ldots, n$ in each cell. In this talk we show, as part of a more general result, that any defining set for the full $n$-Latin square has size $n^3(1 - o(1))$. The full design $N(v, k)$ is the unique simple design with parameters $(v, k, (v-2k)/2)$; that is, the design consisting of all subsets of size $k$ from a set of size $v$. We show that any defining set for the full design $N(v, k)$ has size $\binom{v}{k} (1 - o(1))$ (as $v - k$ becomes large). These results improve existing results and are asymptotically optimal. In particular, the latter result solves an open problem given in (Donovan, Lefevre, et al, 2009), in which it is conjectured that the proportion of blocks in the complement of a full design will asymptotically approach zero.

(Wednesday 09:00)
Degree conditions for partitioning a graph into cycles with a specified number of chords

Shuya Chiba∗, Suyun Jiang and Jin Yan

*Kumamoto University

The classical Ore’s theorem (Amer. Math. Monthly 1960) states that every graph \( G \) of order \( n \geq 3 \) with \( \sigma_2(G) \geq n \) is hamiltonian, where \( \sigma_2(G) \) denotes the minimum degree sum of two non-adjacent vertices in \( G \). Brandt, Chen, Faudree, Gould and Lesniak (J. Graph Theory 1997) generalized the result by showing that the Ore’s degree condition guarantees the existence of a partition of a graph into exactly \( k \) vertex-disjoint cycles for sufficiently large graphs. In this study, we extend this result to a “chorded cycle” version. For a positive integer \( c \), a \( c \)-chorded cycle of a graph \( G \) is a subgraph of \( G \) consisting of a cycle and \( c \) additional edges joining two vertices of the cycle. We give sharp degree conditions for partitioning a graph into exactly \( k \) vertex-disjoint \( c \)-chorded cycles and show that our result is a generalization of the result of Brandt et al.

(Wednesday 09:00)

Permanents of higher dimensional matrices.

Billy Child∗

*Monash University

A higher dimensional matrix of order \( n \) and dimension \( d \) is called 1-polystochastic if it is non-negative and the sum over every line equals 1. Such a matrix that has a single 1 in each line and is otherwise zero is called a 1-permutation matrix. A diagonal of a higher dimensional matrix is a choice of \( n \) entries, no two in the same hyperplane. The 1-permanent of a higher dimensional matrix is the sum over the diagonals of the product of the entries within the diagonal.

For a given order \( n \) and dimension \( d \), the set of 1-polystochastic matrices form a convex polytope that includes the 1-permutation matrices within the set of vertices. For even \( n \) and odd \( d \), we give a construction for a class of 1-permutation matrices with zero 1-permanent. Consequently we show that the set of 1-polystochastic matrices with zero 1-permanent has at least \( (d!)^{-1}n^{n^3/2(1/2-o(1))} \) vertices and dimension \( \Omega(n^{3/2}) \) as \( n \to \infty \).

(Thursday 11:00)
New methods for finding minimum genus embeddings of graphs

Marston Conder*

*University of Auckland

The question of how to find the smallest genus of all embeddings of a given finite connected graph on an orientable (or non-orientable) surface has a long and interesting history. In this talk I will describe some new approaches that can be helpful, in both the orientable and non-orientable cases. One involves taking orbits of subgroups of the automorphism group on cycles of particular lengths in the graph as candidates for subsets of the faces of an embedding. Another uses properties of an auxiliary graph defined in terms of compatibility of these cycles, and others make use of integer linear programming methods. This work was motivated by the problem of finding the minimum genus of the Hoffman-Singleton graph, and succeeded not only in solving that problem but also in answering several other open questions, dating back to 1998. Much of this is joint work with Klara Stokes (Sweden and Ireland).

(Thursday 10:30)

The rank of random matrices over finite fields

Amin Coja-Oghlan and Jane (Pu) Gao*

*Monash University

We determine the rank of a random matrix over a finite field with prescribed numbers of non-zero entries in each row and column. As an application we determine the rate of random low-density parity check codes, which verifies a conjecture of Lelarge. The proofs are based on the Aizenman-Sims-Starr coupling arguments and the interpolation method from mathematical physics. As a byproduct we also determine the size of the 2-core of a random hypergraph with prescribed distributions on the degrees of the vertices and on the sizes of the hyperedges.

(Monday 15:30)
On the Stern–Brocot tree, continued fractions, and Lyndon words

Amy Glen

*Murdoch University

The Stern–Brocot tree is an infinite complete binary tree in which the vertices have a one-to-one correspondence to the positive rational numbers. This tree possesses many intriguing features. Of particular interest is the property that, for each natural number \( n \), the rational numbers corresponding to the vertices in the \( n \)-th row of the tree are precisely those for which the partial quotients in their continued fraction expansion sum to \( n \).

Corresponding to each vertex \( p/q \) in the Stern–Brocot tree (with \( p < q \) and \( \gcd(p,q) = 1 \)) is the so-called Christoffel word of slope \( p/q \), i.e., the balanced Lyndon word over a binary alphabet \( \{a, b\} \) containing \( p \) occurrences of the letter \( b \) and \( q - p \) occurrences of the letter \( a \). (Note that a Lyndon word is a primitive word on an ordered alphabet that is lexicographically less than all of its conjugates.) It is known [1] that the Christoffel word of slope \( p/q = [0; 1 + d_1, d_2, \ldots, d_{m-1}, 1] \) (denoted by \( c_{p/q} \)) contains \( d_1 + d_2 + \cdots + d_{m-1} + 3 \) Lyndon factors. Thus, if \( r(p/q) \) denotes the row number corresponding to the position of \( p/q \) in the Stern–Brocot tree, then the number of Lyndon factors in \( c_{p/q} \) is \( r(p/q) + 1 \).

This talk will focus on some interrelated properties of the Stern–Brocot tree, continued fractions and Christoffel words, and their usefulness in proving a conjecture about finite words that contain the fewest number of Lyndon factors.


Approximately counting independent sets in graphs with bounded bipartite pathwidth

Martin Dyer, Catherine Greenhill* and Haiko Müller

*University of New South Wales, Sydney

In 1989, Jerrum and Sinclair showed that a natural Markov chain for counting matchings in a given graph \( G \) is rapidly mixing. This chain can equivalently be viewed as counting independent sets in line graphs. We generalise their approach to the class of all graphs with the following property: every bipartite induced subgraph of \( G \) has pathwidth at most \( p \). Here \( p \) is a positive integer and the mixing time of the Markov chain will depend on \( p \).

We also describe two classes of graphs (described in terms of forbidden induced subgraphs) that satisfy this condition. Both of these classes generalise the class of claw-free graphs.

(Monday 16:30)
Connectivity functions on cyclically ordered sets

Jasmine Hall*, Charles Semple, Geoff Whittle
*Victoria University of Wellington

Connectivity functions are set functions that generalize connectivity in graphs and matroids. It is known that the theory of branch width and tangles works in this general setting. In this talk I will discuss connectivity functions that are based on a cyclically ordered set and show that the theory of branch-width and tangles extends to such connectivity functions.

(Tuesday 11:00)

A cycle of maximum order in a graph of high minimum degree has a chord

Daniel J. Harvey*
*The University of Western Australia

A long standing conjecture of Thomassen states that every cycle of maximum order in a 3-connected graph contains a chord. While many partial results towards this conjecture have been obtained, the conjecture itself remains unresolved. In this talk, we shall discuss a result for graphs of high minimum degree and show that Thomassen’s conjecture holds in that case. In fact, this result holds for all graphs with high minimum degree, rather than just 3-connected graphs, and is within a constant factor of best possible whenever we make no assumption on the connectivity. The result also generalises to show that large minimum degree forces a large difference between the order of the largest cycle and the order of the largest chordless cycle.

(Friday 10:30)

Defective and Clustered Choosability of Sparse Graphs

Kevin Hendrey* and David R. Wood
*Monash University

An (improper) graph colouring has defect $d$ if each monochromatic subgraph has maximum degree at most $d$, and has clustering $c$ if each monochromatic component has at most $c$ vertices. Given an infinite class of graphs $\mathcal{G}$, it is interesting to ask for the minimum integer $\chi_\Delta(\mathcal{G})$ for which there exist an integer $d$ such that every graph in $\mathcal{G}$ has a $\chi_\Delta(\mathcal{G})$-colouring with defect at most $d$, and the minimum integer $\chi_*(\mathcal{G})$-colouring for which there exist an integer $c$ such that every graph in $\mathcal{G}$ has a $\chi_*(\mathcal{G})$-colouring with clustering at most $c$. We explore clustered and defective colouring in graph classes with bounded maximum average degree. As an example, our results show that every earth-moon graph has an 8-colouring with clustering at most 405. Our results hold in the stronger setting of list-colouring.

(Friday 11:30)
Decompositions of complete multipartite graphs into Hamilton paths

Darryn Bryant, Hao Chuien Hang and Sara Herke*

*The University of Queensland

If a graph with $n$ vertices and $m$ edges can be decomposed into edge-disjoint Hamilton paths, then $t = \frac{m}{n-1}$ is an integer (equal to the number of paths) and the maximum degree is at most $2t$, because each Hamilton path has maximum degree 2. We recently proved that for complete multipartite graphs, these necessary condition are also sufficient. In this talk we will discuss some of techniques that were used in our proof.

(Thursday 11:00)

From topological to finite geometries and back

Duy Ho*

*University of Canterbury

In 1990, Grundhöfer showed that, under suitable assumptions, a topological projective plane can be written as an inverse limit of finite discrete incidence structures. He also showed that every finite projective plane is a continuous epimorphic image of some topological projective plane. This type of results were then extended to generalised quadrangles and hexagons. In this talk we discuss how this approach can be applied to Möbius, Laguerre and Minkowski planes.

(Thursday 16:30)

Induced path numbers of regular graphs

Saieed Akbari, Daniel Horsley* and Ian Wanless

*Monash University

The path cover number of a graph $G$, the smallest size of a collection of paths in $G$ such that every vertex of $G$ is in exactly one of the paths, has been very well studied. The induced path number, the analogous quantity when we also demand that the paths be induced, has also received some attention. This talk will discuss some bounds on the induced path number of connected regular graphs that Saieed Akbari, Ian Wanless and I established recently, focussing on the cubic case.

(Wednesday 10:30)
Characterizations of hyperplanes related to quadrics in projective space

Alice Man Wa Hui*, Susan Barwick and Wen-Ai Jackson
*BNU-HKBU United International College

There are a number of known characterisations of subspaces relating to quadrics in the $n$-dimensional projective space $\text{PG}(n,q)$. In this talk, we will provide a survey on those relating to the non-singular quadric $Q(4,q)$ in $\text{PG}(4,q)$. For even $q$, we will give a combinatorial characterisation of the set of hyperplanes that meet $Q(4,q)$ in an elliptic quadric, and a characterisation of those meeting in a hyperbolic quadric.

(Thursday 15:30)

Construction of new Griesmer codes of dimension 5

Yuto Inoue* and Tatsuya Maruta
*Osaka Prefecture University

An $[n, k, d]_q$ code is a linear code of length $n$, dimension $k$ and minimum weight $d$ over the field of $q$ elements. We construct $[n, 5, d]_q$ codes attaining the Griesmer bound for $2q^4 - 3q^3 + 1 \leq d \leq 2q^4 - 3q^3 + q^2$ and for $3q^4 + 5q^3 + 1 \leq d \leq 3q^4 + 5q^3 + q^2$ for every $q \geq 3$ using some geometric methods such as projective dual and geometric puncturing.

(Wednesday 09:30)

The sandwich conjecture of random regular graphs and more

P. Gao, Mikhail Isaev*, B.D. McKay
*Monash University

The sandwich conjecture formulated in [Kim and Vu, Advances in Mathematics, 2004] states that if $\min\{d, n - d\} \geq \log n$, then the random $d$-regular graph on $n$ vertices, denoted by $R(n,d)$, can asymptotically almost surely be “sandwiched” between $G(n, p_1)$ and $G(n, p_2)$ where probabilities $p_1$ and $p_2$ are both $(1 + o(1))d/n$. They proved this conjecture for the range $\log n \leq d \leq n^{1/3-o(1)}$ with a defect in one side of sandwiching: a few edges from each vertex should be deleted from the random regular graph to guarantee the containment of the remaining graph in $G(n, p_2)$. Recently, their result (one-sided containment $G(n, p_1) \subseteq R(n,d)$) was improved by Dudek, Frieze, Ruciński and Šileikis to $d = o(n)$.

We prove the sandwich conjecture (with perfect containments on both sides) for all values of $d$ such that $d \gg n/\log n$. For $d = O(n/\log n)$, we show a weaker version of the sandwich conjecture, where $p_2$ is approximately $(d/n) \log n$.

(Monday 16:00)
Empowering the Configuration-IP — New PTAS Results for Scheduling with Setups Times

Klaus Jansen*, Kim-Manuel Klein, Marten Maack and Malin Rau

*University of Kiel

Integer linear programs of configurations, or configuration IPs, are a classical tool in the design of algorithms for scheduling and packing problems, where a set of items has to be placed in multiple target locations. Herein a configuration describes a possible placement on one of the target locations, and the IP is used to choose suitable configurations covering the items. We give an augmented IP formulation, which we call the module configuration IP. It can be described within the framework of n-fold integer programming and therefore be solved efficiently using the algorithm by Hemmecke, Onn, and Romanchuk. As an application, we consider scheduling problems with setup times, in which a set of jobs has to be scheduled on a set of identical machines, with the objective of minimizing the makespan. For instance, we investigate the case that jobs can be split and scheduled on multiple machines. However, before a part of a job can be processed an uninterrupted setup depending on the job has to be paid. For both of the variants that jobs can be executed in parallel or not, we obtain an efficient polynomial time approximation scheme (EPTAS) of running time $f(1/\epsilon) \times \text{poly}(|I|)$ with a single exponential term in $f$ for the first and a double exponential one for the second case. Previously, only constant factor approximations of $5/3$ and $4/3 + \epsilon$ respectively were known. Furthermore, we present an EPTAS for a problem where classes of (non-splittable) jobs are given, and a setup has to be paid for each class of jobs being executed on one machine.

(Monday 15:30)

Properties of Powerful Sets and the Rank Function

Ben Jones*, G. Farr and K. Morgan

*Monash University

Powerful sets were introduced by Farr & Wang (2017). Although powerful sets include the cocircuit spaces of binary matroids, almost all powerful sets are non-linear. Each powerful set can be assigned a non-negative integer valued rank function analogous to the matroid rank, although it does not in general satisfy the matroid rank axioms. In this talk, we present results on the structural properties of powerful sets and their rank functions.

(Tuesday 11:30)
Well-Quasi-ordering in Lattice Path Presentations

Meenu Mariya Jose* and Dillon Mayhew
*Victoria University of Wellington

Consider a pair of lattice paths \( P \) and \( Q \) that run from the origin to \((m, r)\) such that each step is a unit in the direction North or East, where \( P \) never goes above \( Q \). Such a structure is called a lattice path presentation. For each such presentation, there is an associated matroid, introduced by Bonin, de Mier and Noy [1]. Notions of deletion, contraction, and minors exist for lattice path presentations, just as they do for graphs and matroids.

A quasi-order is a relation that is reflexive and transitive. A well-quasi-order is a quasi-ordering \( \leq \), with the property that if \( a_0, a_1, \ldots \) is an infinite sequence, then there exists \( i \) and \( j \) such that \( a_i \leq a_j \). The gigantic Graph Minors project, that took Robertson and Seymour more than two decades to complete, proved that graphs are well-quasi-ordered under the minor relation. Lattice path presentations are not well-quasi-ordered under the minor relation. However, if we bound the size of any square contained in the presentation, then the class becomes well-quasi-ordered. This is equivalent to bounding the branch-width of the associated matroid. The proof of the theorem uses the elegant minimal bad sequence argument that Nash-Williams [2] employs to prove that finite trees are well-quasi-ordered.

References


Unavoidable Minors of 4-Connected Graphs and 4-Connected Binary Matroids

Susan Jowett* and Geoff Whittle
*Victoria University of Wellington

Ramsey’s theorem tells us, loosely speaking, that if we have a large enough graph we can find a large complete graph or a large set of disjoint vertices as a minor. These are the unavoidable minors of a graph. We also know the unavoidable minors of 2, 3 and 4-connected graphs. We will look at the progress on finding the unavoidable minors of 4-connected matroids and give a broad sketch of some of the proof techniques.

(Tuesday 15:30)
Semipaired Domination in Maximal Outerplanar Graphs

M. A. Henning and Pawaton Kaemawichanurat*
*King Mongkut’s University of Technology Thonburi

A subset $S$ of vertices in a graph $G$ is a dominating set if every vertex in $V(G) \setminus S$ is adjacent to a vertex in $S$. If the graph $G$ has no isolated vertex, then a semipaired dominating set of $G$ is a dominating set of $G$ with the additional property that the set $S$ can be partitioned into two element subsets such that the vertices in each subset are at most distance two apart. The semipaired domination number $\gamma_{pr2}(G)$ is the minimum cardinality of a semipaired dominating set of $G$. Let $G$ is a maximal outerplanar graph of order $n$ with $n_2$ vertices of degree 2. We show that if $n \geq 5$, then $\gamma_{pr2}(G) \leq \frac{2}{5}n$. Further, we show that if $n \geq 3$, then $\gamma_{pr2}(G) \leq \frac{1}{3}(n + n_2)$. Both bounds are shown to be tight.

Finding monochromatic combinatorial lines with few intervals

David Conlon, Nina Kamčev* and Christoph Spiegel
*Monash University

The Hales-Jewett Theorem states that any $r$-colouring of $[m]^n$ contains a monochromatic combinatorial line if $n$ is sufficiently large. Shelah’s proof of the theorem for $m = 3$ yields a monochromatic combinatorial line in $[3]^n$ whose set of ‘active coordinates’ is the union of at most $r$ intervals. The question of how optimal this structure is has been investigated by Conlon, Kamčev, Leader, Räty and Spiegel, revealing a surprising alternation depending on the parity of $r$. Namely, for odd $r$, there are colourings avoiding all monochromatic lines whose wildcard set consists of fewer than $r$ intervals. However, for even $r$ and large $n$, we can achieve $r - 1$ intervals.

In this talk, we will outline some of the ideas involved in the proofs, focusing on the upper bound for even $r$. We hope that our approach could also be relevant to the corresponding problem for larger alphabets.

A new extension theorem for ternary linear codes

Hitoshi Kanda* and Tatsuya Maruta
*Osaka Prefecture University

An $[n, k, d]_q$ code is a linear code of length $n$, dimension $k$ and minimum weight $d$ over the field of $q$ elements. An $[n, k, d]_q$ code $C$ is called extendable if $C$ can be extended to an $[n+1, k, d+1]_q$ code. We give a new extension theorem for ternary linear codes. As an application, we prove the nonexistence of a $[512, 6, 340]_3$ code, which is a new result.
The agreement distance of phylogenetic networks

Jonathan Klawitter∗
∗University of Auckland

The minimal number of rooted subtree prune and regraft (rSPR) operations needed to transform one phylogenetic tree into another one induces a metric on phylogenetic trees – the rSPR-distance. The rSPR-distance between two phylogenetic trees $T$ and $T'$ can be characterised by a maximum agreement forest; a forest with a minimal number of components that covers both $T$ and $T'$. The rSPR operation has recently been generalised to phylogenetic networks with, among others, the prune and regraft (PR) operation. In this talk, we introduce maximum agreement graphs of two phylogenetic networks as a generalisation of maximum agreement forests. We then look at the relation of the PR-distance and the agreement distance – the metric induced by maximum agreement graphs.

(Monday 11:00)

Graphs with the maximum value of Graovac-Pisanski index

Martin Knor*, Riste Škrekovski and Aleksandra Tepeh
*Slovak University of Technology in Bratislava

Graovac-Pisanski index of a graph $G$ is defined as

$$GP(G) = \frac{|V(G)|}{2|Aut(G)|} \sum_{u \in V(G)} \sum_{\alpha \in Aut(G)} dist_G(u, \alpha(u)),$$

where $Aut(G)$ is the group of automorphisms of $G$. For vertex-transitive graphs Graovac-Pisanski index coincides with the Wiener index. While Wiener index is correlated with boiling points of alcanes, Graovac-Pisanski index is correlated with melting points of hydrocarbon molecules. Obviously, if the group of automorphisms is trivial, then $GP(G) = 0$. Interesting is the opposite problem. We proved that if $T$ is a tree on $n$ vertices with the maximum value of Graovac-Pisanski index, $n \geq 8$, then $T$ is either a path on $n$ vertices $P_n$, or a graph obtained from $P_{n-4}$ by attaching two pendant vertices to each end of the path. Moreover, we proved that if $G$ is a unicyclic graph on $n$ vertices with the maximum value of Graovac-Pisanski index, then $G$ is the $n$-cycle.

(Tuesday 11:00)

A family of hemisystems on the parabolic quadric.

Jesse Lansdown* and Alice Niemeyer
*University of Western Australia

A parabolic quadric, $Q(2d, q)$, of rank $d$ is given by taking the totally singular subspaces of a $(2d+1)$-dimensional vector space over $\mathbb{F}_q$ under a quadratic form. The largest such subspaces are called maximals, while the 1-spaces are called points. A hemisystem $\mathcal{H}$ of $Q(2d, q)$ is a set of maximals such that exactly half of the maximals on every point are contained in $\mathcal{H}$. Recently, Cossidente and Pavese constructed a family of hemisystems for $Q(6, q)$, $q$ odd[1]. In this talk I will present a new family of hemisystems for $Q(2d, q)$, $q$ odd, and $d \geq 2$.

(Wednesday 10:30)
A Stallings’ type theorem for quasi-transitive graphs

Matthias Hamann, Florian Lehner*, Babak Miraftab, and Tim Rühmann

*University of Warwick

By a well known theorem of Stallings, any finitely generated group with more than one end is either a free product with amalgamation or an HNN extension over a finite subgroup. In this talk, we discuss an analogous result for quasi-transitive graphs.

(Thursday 15:30)

Partial projections are past-finite-universal

Erkko Lehtonen and Nareupanat Lekkoksung*

*Khon Kaen University

We investigate the structure of the minor poset of partial functions. A poset is past-finite-universal if it admits an embedding of every countable poset whose principal ideals are finite and it itself is such a poset. M. Couceiro and M. Pouzet showed that the minor poset of Boolean functions is past-finite-universal. This property carries over to minor posets of partial functions on finite sets. We improve this result by showing that certain small fragments of this poset (e.g., partial projections, constant functions) are past-finite-universal.

(Tuesday 16:00)
Gastineau-Hills’ quasi-Clifford algebras and plug-in constructions for Hadamard matrices

Paul Leopardi

∗Australian Bureau of Meteorology. The University of Melbourne.

The quasi-Clifford algebras, and their Wedderburn structure and representation theory, as described by Gastineau-Hills in 1980 and 1982, should be better known, and have only recently been rediscovered.

These algebras and their representation theory provide effective tools to address certain questions relating to plug-in constructions for Hadamard matrices.

The key question addressed is: Given \( \lambda \), a pattern of amicability / anti-amicability, with \( \lambda_{j,k} = \lambda_{k,j} = \pm 1 \), find a set of \( n \) monomial \( \{-1, 0, 1\} \) matrices \( D \) of minimal order such that

\[
D_j D^T_k - \lambda_{j,k} D_k D^T_j = 0 \quad (j \neq k).
\]

The quasi-Clifford algebras provide a solution to this question via their irreducible real monomial representations.

This is essentially the talk given in July at AGACSE 2018 in Campinas, Brazil, and repeated the following week at the Symposium on Clifford Algebras, Mathematical Physics and Related Topics, at the Universidade Federal do ABC, Brazil. See https://arxiv.org/abs/1804.09454 (Thursday 16:30)

Uniquely Wilf Permutation Classes

Michael Albert and Jinge Li

∗University of Otago

A permutation class \( C \) is said to be uniquely Wilf, if for every integer \( k \) and all \( n > k \), each permutation of size \( k \) belonging to \( C \) is contained in the same number of permutations of size \( n \) that belong to \( C \). We identify all uniquely Wilf classes that avoid a pattern of size 3. Additionally, we show that for classes that avoid no pattern of size 3 or less, there exist no uniquely Wilf-classes having three or more basis elements of size 4. The methodology used is to cast some of the necessary conditions that uniquely Wilf classes must satisfy as constraint satisfaction problems (CSPs). Computational CSP solvers are then used to rule out many possibilities. The remaining candidates can then be examined directly and proven to be uniquely Wilf.

(Monday 16:30)
Tight Kernels in Phylogenetics

Steven Kelk and Simone Linz*

*University of Auckland

Kernelization is a powerful technique from parameterized complexity that preprocesses a problem instance by shrinking the input to its core. A subsequent exhaustive search often performs significantly faster than without preprocessing. To achieve kernelization results in phylogenetics (and other application areas), data reduction rules are frequently used. For example, to compute the so-called tree-bisection and reconnection (TBR) distance between two phylogenetic trees, the subtree and chain reduction have been developed. By applying these two reductions repeatedly to a pair of phylogenetic trees until no more reduction is possible, it can be shown that the number of leaves of the two reduced trees is linear in the TBR distance of the two original (unreduced) trees. Variations of the subtree and chain reduction have also been used to calculate other tree distances that are of interest in searching the space of all phylogenetic trees. In this talk, we reanalyze a number of kernelization results for phylogenetic trees and present improved kernels that are tight.

(Monday 11:30)

Tilings of the Sphere by Almost Equilateral Pentagons

Yohji Akama and Hoi Ping Luk* and Min Yan

*The Hong Kong University of Science & Technology

The classification of edge-to-edge tilings of the sphere by congruent pentagons can be divided into three cases: variable edge lengths case, equilateral case, and almost equilateral case. The first two cases have been largely settled by Min Yan and his collaborators. The almost equilateral case (four edges of equal length, the fifth of different length) is the most difficult, and earlier techniques are insufficient. We introduced decision-making algorithms in wxMaxima and some new geometric constraints for the almost equilateral case. We obtained full classification for three distinct angles and partial results for five distinct angles. Alongside the fundamental tilings - Earth Map Tilings, we have discovered some interesting tilings not seen in the other pentagon cases. We will also discuss the linkage between Earth Map Tilings and the geometric realisation of the dual graphs to \{((a, b), k}\}-spheres.

(Wednesday 09:30)
Generalisations of the Erdős-Ko-Rado Theorem for permutations

Adam Mammoliti*

*University of New South Wales, Sydney

The celebrated Erdős-Ko-Rado Theorem states that if \( n \geq 2k \) and \( \mathcal{F} \) is a family of \( k \)-subsets of \([n]\) such that \( A \cap B \neq \emptyset \) for all sets \( A, B \in \mathcal{F} \), then \( |\mathcal{F}| \leq \binom{n-1}{k-1} \), with equality for \( n > 2k \) occurring precisely when \( \mathcal{F} \) is the family of all \( k \)-subsets containing a fixed element of \([n]\). Since its discovery, the Erdős-Ko-Rado Theorem has been generalised extensively and analogous results have been shown for structures other than sets. In particular, an analogue of the Erdős-Ko-Rado Theorem has been shown for families of permutations of \([n]\).

In this talk, I will give an overview of existing and new analogues of the Erdős-Ko-Rado Theorem for permutations and a variety of structures that generalise permutations. We will present a new and natural framework from which the Erdős-Ko-Rado Theorem analogues for each of these structures can be derived. We see from this framework that each of the existing analogues directly generalise the Erdős-Ko-Rado Theorem in a straightforward way. I will also present new results that show how to further generalise the Erdős-Ko-Rado Theorem analogues provided by the framework, using only relatively simple inductive arguments.

(Wednesday 09:00)

Balanced Equi-\( n \)-squares

Saieed Akbari, Trent G. Marbach*, Rebecca J. Stones, and Zhuanhao Wu

*Nankai University

In this presentation, we show the recent work that has been undertaken to understand \( d \)-balanced equi-\( n \)-squares. With the requirement that \( d \) is a divisor of \( n \), these structures are \( n \times n \) matrices containing symbols from \( \mathbb{Z}_n \) in which any symbol that occurs in a row or column, occurs exactly \( d \) times in that row or column. There are connections with Latin square of order \( n \) that decompose into \( d \times (n/d) \) subrectangles, which we exploit to construct \( d \)-balanced equi-\( n \)-squares. We also show connections with \( \alpha \)-labellings of graphs, which enables us to both construct new \( d \)-balanced equi-\( n \)-squares and construct new \( \alpha \)-labellings of graphs.

(Monday 16:00)
The intersection of longest paths in generalised theta graphs

Sarah Mark*, Jeanette McLeod, and Brendan McKay
*University of Canterbury, New Zealand

In 1966, Gallai asked whether all of the longest paths in a connected graph share a vertex. It is known that every pair of longest paths share a vertex, and that there are graphs in which some seven or more longest paths do not share a vertex, but the question remains open for three to six longest paths. The answer is also known to be positive for certain classes of graphs. We have proved that the answer is positive for a class of graphs consisting of a generalisation of a theta graph, and are investigating how this method can be applied to other graphs.

(Tuesday 16:00)

On the non-trivial blocking sets in binary projective spaces

Nanami Bono, Tatsuya Maruta*, Keisuke Shiromoto and Kohei Yamada
*Osaka Prefecture University

We denote by PG($r, q$) the projective geometry of dimension $r$ over the field of $q$ elements. A $j$-flat is a projective subspace of dimension $j$ in PG($r, q$). A set of points in PG($r, q$) meeting every $(r - k)$-flat is called a $k$-blocking set or a blocking set with respect to $(r - k)$-flats. A $k$-flat in PG($r, q$) is the smallest $k$-blocking set, and a blocking set containing a $k$-flat in PG($r, q$) is called trivial. A $k$-blocking set $B$ is minimal if $B \setminus \{P\}$ is no longer a $k$-blocking set for any point $P$ of $B$. It is known that every non-trivial minimal 1-blocking set in PG($3, 2$) is an elliptic quadric. We classify the $k$–blocking sets in PG($4, 2$) for $k = 1, 2, 3$. We also consider the non-trivial minimal 1-blocking sets in PG($r, 2$) for $r \geq 5$.

(Tuesday 15:30)
Courcelle’s Theorem, tree automata, and pigeonhole classes of matroids

Daryl Funk, Dillon Mayhew*, Mike Newman, and Geoff Whittle

*Victoria University of Wellington

We think of a tree automaton as processing a binary tree, from leaves to root, applying a state to each node. The computation is initialised by applying states to the leaf nodes. The state that is applied to any non-leaf node depends on the states applied to its children. The automaton halts when it applies a state to the root, and depending on that state, we say that the automaton either accepts or rejects. The initial states can encode a subset of the leaves. In this case, we think of the automaton as either accepting or rejecting each subset of the leaves. Thus each tree automaton gives rise to a family of hypergraphs. The vertex set of such a hypergraph is the set of leaves of a tree, and a subset of leaves is a hyperedge if and only if it is accepted by the automaton. We have characterised the families of hypergraphs that arise in this way.

Our interest in tree automata arises from Courcelle’s Theorem for graphs and Hliněný’s analogous theorem for matroids. Courcelle’s Theorem tells us that we can test certain hard graphic properties (such as Hamiltonicity and 3-colourability) in polynomial time if we restrict the input to graphs with bounded structural complexity (in particular, bounded tree-width). Hliněný proved an analogue of Courcelle’s Theorem for matroids representable over finite fields. We use our results on tree automata to extend Hliněný’s Theorem further, to additional classes of matroids.

Little background knowledge of automata or matroids will be assumed in this talk.

(Thursday 11:00)

Feasible bases for a polytope related to the Hamilton cycle problem

Sogol Mohammadian* and Thomas Kalinowski

*University of Newcastle

About 20 years ago, Filar and Krass proposed an approach to the Hamilton cycle problem which is based on the theory of Markov decision processes. This initiated the study of various closely related polytopes constructed from an input graph $G$, all of which have the property that the Hamilton cycles of $G$ correspond to certain extreme points of the polytope. In particular, one can use a random walk on the set of all the extreme points in order to search for Hamilton cycles. To analyse the efficiency of this approach it is necessary to understand the combinatorial structure of the polytopes, in particular (1) the prevalence of the extreme points corresponding to Hamilton cycles within the set of all extreme points, and (2) the mixing properties of the used random walk. Towards this goal, we present some recent results on the structure of the feasible bases for a particular polytope introduced in 2011 by Eshragh and Filar.

(Tuesday 16:30)
Large structured induced subgraphs

Marthe Bonamy, František Kardoš, Tom Kelly, Peter Nelson

*University of Waterloo

Given a graph $G$ on $m$ edges and a class $\mathcal{H}$ of graphs, what is the smallest $\alpha$ such that there is a set $D$ of at most $\alpha m$ vertices of $G$ for which $G - D \in \mathcal{H}$? We discuss new results on this question when $\mathcal{H}$ is one of the classes of forests, linear forests, pseudoforests and the $K_t$-free graphs for a fixed $t \geq 2$.

(Thursday 10:30)

Efficiently computing graph decompositions

William Pettersson

*University of Glasgow

Many problems in combinatorics can easily be solved by finding a sufficient set of “building blocks” — partial combinatorial structures which, when joined together correctly, can create all required solutions. The difficulty lies in finding this sufficient set, as it often lacks any symmetry or regularity that makes finding combinatorial structures easy. In this talk I will discuss various computational approaches that can be utilised to find such a set, with practical examples for graph decomposition problems. Four approaches in particular will be discussed: brute force, bespoke, integer programming and constraint programming.

This talk aims to present techniques rather than results, and is suitable for anyone looking to utilise computational approaches in combinatorial research.

(Thursday 11:30)
Cubic Graphs with few or many Hamilton Cycles

Gordon Royle* and Irene Pivotto

*University of Western Australia

There is a large body of work regarding the existence of Hamilton cycles in cubic graphs, in particular what combinations of graph-theoretical properties guarantee that a cubic graph with those properties is hamiltonian.

Here we consider instead the question of the enumeration of Hamilton cycles in various families of hamiltonian cubic graphs. For example, it is well-known that a cubic hamiltonian graph on $2n$ vertices cannot have fewer than three Hamilton cycles (independent of $n$), but a corresponding upper bound is not known (though there is a very convincing conjecture).

Chia and Thomassen studied a number of families of cubic graphs with particular connectivity constraints. They posed several questions / conjectures regarding both upper and lower bounds for the numbers of Hamilton cycles in these families and the graphs meeting the bounds.

In this talk, I will describe the progress made (in joint work with Irene Pivotto) towards answering some of their questions. A substantial fraction of this involves developing bespoke techniques for counting Hamilton cycles in specific infinite families of cubic graphs.

(Thursday 16:30)

The Critical Problem for Binary Matroids

Keisuke Shiromoto*

*Kumamoto University

Let $\mathbb{F}_q$ be a finite field of $q$ elements. For any subset $S \subseteq \mathbb{F}_q^n$, define the critical exponent of $S$ as follows:

$$c(S, q) := n - \max\{ r \in \mathbb{Z}^+ : \text{there exists an } r\text{-dimensional subspace } D \text{ of } \mathbb{F}_q^n \text{ with } D \cap S = \emptyset \}.$$ 

This number was introduced in the context of matroid theory where it has attracted attention as the critical exponent $c(M, q)$ of an $\mathbb{F}_q$-representable matroid $M$. An $\mathbb{F}_q$-representable matroid is a matroid obtained by a set of vectors in $\mathbb{F}_q^n$. Thus we identify an $\mathbb{F}_q$-representable matroid $M$ as a subset $S \subseteq \mathbb{F}_q^n$ in this talk.

The Critical Problem (Crano and Rota, 1970) is the problem of finding the critical exponent of a fixed subset in a vector space over a finite field, or in the original statement of the problem, of finding the least number of hyperplanes whose intersection contains no element of the subset.

In this talk, we mainly focus on the problem for binary matroids. In particular, we will discuss the Walton-Welsh Conjecture (1980) from the perspective of blocking sets in binary projective spaces.

This talk is a part of my joint work with Tatsuya Maruta.

(Thursday 11:30)
Palindromes in Starlike Trees

Amy Glen, Jamie Simpson* and Bill Smyth
*Curtin University

A palindrome is a factor of a word which equals its own reverse. The word $abaababa$ contains the distinct palindromes $a$, $b$, $aa$, $aba$ and four others. It is well known that a word of length $n$ can contain at most $n$ distinct palindromes. Words can be induced by paths in edge-labeled graphs which raises the question, “for a given graph, what is the maximum number of distinct palindromes that can be induced by edge-labeling?”. For a length $n$ cycle this is known to be less than $5n/3$. In this talk we will consider the case of an edge-labeled starlike tree, that is, a graph consisting of several equal length paths attached to a single root vertex.

(Monday 10:30)

Defective 2-colorings of planar graphs without 4-cycles and 5-cycles

Pongpat Sittitrai* and Kittikorn Nakprasit
*Khon Kaen University

A graph $G$ is called $(d_1,d_2,\ldots,d_k)$-colorable if the set of vertices can be partitioned in to $k$ sets $V_1, V_2, \ldots, V_k$ such that the induced subgraph $G[V_i]$ for $i \in [k]$ has the maximum degree at most $d_i$. This notion generalizes those of proper $k$-coloring (when $d_1 = d_2 = \cdots = d_k = 0$).

In this talk, we provide two results of a planar graph without 4-cycles and 5-cycles $G$ with the followings:

1. We construct non-$(1,k)$-colorable planar graphs without 4-cycles and 5-cycles for every positive integer $k$.
2. We prove that $G$ is $(d_1,d_2)$-colorable where $(d_1,d_2) = (4,4), (3,5)$, and $(2,9)$.

(Monday 11:00)

Uniform generation of random Latin rectangles

Pu Gao, Angus Southwell*, and Nick Wormald
*Monash University

We present an algorithm which generates a random $d \times n$ Latin rectangle uniformly at random. The expected running time of the algorithm is $O(d^3n)$ when $d = o(\sqrt{n})$. This is a specific case of an algorithm designed for generating sets of non-intersecting matchings uniformly at random.

(Monday 15:30)
The known finite Minkowski planes — a characterization in terms of Klein–Kroll types

Günter Steinke

*University of Canterbury, New Zealand

Minkowski planes are incidence geometries with points, lines (also called generators) and blocks (normally called circles). They are extensions of affine planes by a family of hyperbolic ovals. A finite Minkowski plane of order \( n \) is equivalent to a sharply 3-transitive set of permutations of degree \( n + 1 \). All known finite Minkowski planes have order a prime power \( q \) and correspond to \( PSL(2,q) \cup (PGL(2,q) \setminus PSL(2,q))\alpha \) where \( \alpha \) is an automorphism of the Galois field of order \( q \).

In 1989 Monica Klein and H.-J. Kroll investigated the set of all generators \( G \) for which the group of all \( G \)-translations in a given group \( \Gamma \) of automorphisms of a Minkowski plane \( \mathcal{M} \) is transitive on \( C \setminus G \) where \( C \) is any circle. Here a \( G \)-translation of \( \mathcal{M} \) is an automorphism of \( \mathcal{M} \) that either fixes precisely the points of the generator \( G \) or is the identity; it induces a translation in the derived affine plane of \( \mathcal{M} \) at any point of \( G \). Klein and Kroll obtained a list of six feasible configurations for this set of generators, called the type of \( \Gamma \), labelled A through to F. (This is similar to the Lenz-Barlotti classification for projective planes.) There are examples of groups of automorphisms of (certain) Minkowski planes for each of the six types.

In this talk we review what is known about the types of the full automorphism groups of Minkowski planes. We show that type E cannot occur as the type of a finite Minkowski plane and that type F characterizes the known finite Minkowski planes.

(Thursday 16:00)

Computing the autotopism group of a partial Latin rectangle

Rebecca J. Stones*, Raúl M. Falcón, Daniel Kotlar, and Trent G. Marbach

*Nankai University

Computing the autotopism group (the stabilizer subgroup under permutations of the rows, columns, and symbols) of a partial Latin rectangle (PLR) can be performed in a variety of ways. We experimentally compare various methods and identify the design goals one should have in mind when designing software for computing autotopism groups of partial Latin rectangles.

(Thursday 15:30)
An Online Graph Atlas

Srinibas Swain* and Graham Farr and Kerri Morgan and Paul Bonnington

*Monash University

In this talk we introduce a new research tool, an online interactive repository of graphs called the Online Graph Atlas (OLGA). The repository is designed to enable efficient retrieval of information by allowing search queries by graphs along with combinations of standard graph parameters. Parameters include chromatic number, chromatic index, domination number, independence number, clique number, matching number, vertex cover number, size of automorphism group, vertex connectivity, edge connectivity, eigenvalue, treewidth, genus.

Among the many online and print graph repositories, Read and Wilson’s book, *An Atlas of Graphs* (OSP, 1998) is one of the most widely used. Inspired by their attempt, Barnes, Bonnington and Farr developed the first prototype of OLGA in 2009, and subsequently extended by Sio, Farr and Bonnington in 2010 by adding more parameters. The current version of OLGA stores over 20 standard parameters for each graph of up to 11 vertices. We used recursive algorithms and exact algorithms for parameter computation. OLGA is not limited to the role of search engine for graphs. We demonstrate how to use OLGA as a tool to explore conjectures and theorems involving the computed parameters.

(Thursday 10:30)

Prolific patterns on combinatorial structures

Michael Albert and Murray Tannock*

*University of Otago

A combinatorial structure, \( w \), in some class, \( C \), is said to be \( p \)-prolific for some substructure, \( p \), if the addition of any minimal element of \( C \) to \( w \) creates at least one new occurrence of \( p \). The set of non-\( p \)-prolific structures forms a downward closed set within the class \( C \). Some substructures have no \( p \)-prolific structures, whereas some structures are prolific for themselves.

We will examine prolificity within the context of integer compositions, presenting necessary conditions on \( p \) for the existence of \( p \) prolific compositions, as well as necessary conditions for a composition to be \( p \)-prolific. We will also present algorithmic methods for finding the minimal \( p \)-prolific compositions in the set of all compositions.

(Monday 16:00)
Cubic vertex-transitive graphs and automorphisms with long orbits.

Primož Potočnik and Micael Toledo∗
∗University of Primorska, IMFM

Let Γ be a graph of order n and let ϕ be an automorphism of Γ. For a fixed integer k, we say that ϕ has a long orbit if at least \( \frac{n}{k} \) vertices lie in the same orbit under the action of the cyclic group \( \langle \varphi \rangle \). We give a general overview of cubic vertex-transitive graphs that admit an automorphism with a long orbit and we completely classify such graphs when \( k = 3 \).

(Thursday 11:30)

The number of directions determined by a function over a finite field

Geertrui Van de Voorde* and Jan De Beule
∗University of Canterbury

If \( f : \mathbb{F} \rightarrow \mathbb{F} \) is a function, then how many directions are determined by the graph of \( f \)? This question was studied intensively for the finite field \( \mathbb{F}_q \), by Ball, Blokhuis, Brouwer, Storme, and Szőnyi, building on the work of Rédei. Their work was partially motivated by its connection with blocking sets in \( \text{PG}(2,q) \).

In recent work, we solved a related problem: we restrict the domain of \( f \) to a subspace of \( \mathbb{F}_q \) (over some subfield \( \mathbb{F}_s \) of \( \mathbb{F}_q \)) and restrict \( f \) to \( \mathbb{F}_s \)-linear maps. In this talk, I will introduce these notions and show the consequences of our theorem for the study of linear sets, which are a central object in finite geometry.

(Tuesday 16:00)

The Down Under Shuffle

Benjamin Cook, Borislav Karaivanov and Tzvetalin S. Vassilev∗
∗Nipissing University

We consider the following way of shuffling of a deck of \( n \geq 1 \) cards. The deck is in hand, and we repeatedly put the top card of the deck in hand under the deck (in hand), then put the top card of the deck in hand on the table (forming a new deck), then put the top card of the deck in hand under the deck (in hand), then put the top card of the deck in hand on top of the deck on the table, ... until the cards in hand are exhausted.

We investigate the properties of this shuffle, considering the following questions:

- Where does the \( k \)th card in the deck end up after \( m \) shuffles?
- What was the original position of the card that is in position \( k \) after \( m \) shuffles?
- How many shuffles does it take to restore the deck in its original order?
This work is motivated by problem number 12008 published in American Mathematical Monthly in December 2017.

(Tuesday 11:30)

### Edge-transitive graphs of small order

**Gabriel Verret** and **Marston Conder**

*University of Auckland*

A graph is *edge-transitive* if, for every two edges, there is an automorphism of the graph mapping one to the other. Motivated by a question of Brendan McKay, we developed a method for finding all edge-transitive graphs of small order. I will briefly explain this method and discuss the results we obtained with it. (We found all edge-transitive graphs of order up to 47, and all bipartite edge-transitive graphs of order up to 63.) I will also explain how some of these graphs helped us answer a 1967 question of Folkman about regular edge-transitive graphs of large valency.

(Tuesday 10:30)

### General Multiplicative Zagreb Indices of Trees

**Tomáš Vetrík** and **Selvaraj Balachandran**

*University of the Free State, Bloemfontein, South Africa*

The first general multiplicative Zagreb index of a graph $G$ is defined as $P^1_a(G) = \prod_{v \in V(G)} (d(v))^a$ and the second general multiplicative Zagreb index of $G$ is $P^2_a(G) = \prod_{v \in V(G)} (d(v))^{ad(v)}$, where $a \neq 0$ is a real number, $V(G)$ is the vertex set of $G$ and $d(v)$ is the degree of a vertex $v \in V(G)$. We present sharp upper and lower bounds on the general multiplicative Zagreb indices for trees.

(Monday 11:30)
Combinatorial views on persistent characters

Kristina Wicke* and Mareike Fischer

*University of Greifswald

In biology, phylogenetic trees are used to represent the evolutionary history and relationships of different species. Often these trees are reconstructed from discrete characters (e.g. DNA data) via character-based tree estimation methods. Here, we consider so-called persistent binary characters. A binary character is called persistent if it can be realized on a tree by at most one $0 \rightarrow 1$ transition followed by at most one $1 \rightarrow 0$ transition. We begin by characterizing these characters in terms of the Maximum Parsimony criterion for tree reconstruction and the famous Fitch algorithm. We then turn to counting persistent characters and establish a relationship between the balance of a tree and its number of persistent characters. We conclude by providing an upper bound on the number of characters together with their persistence status that are needed in order to uniquely determine a tree.

(Monday 10:30)

Higher Dimensional Lattice Walks: Connecting Combinatorial and Analytic Behavior

Stephen Melczer and Mark C. Wilson*

*University of Auckland

We consider the enumeration of walks on the non-negative lattice $\mathbb{N}^d$, with steps defined by a set $S \subset \{-1, 0, 1\}^d \setminus \{\mathbf{0}\}$. Previous work in this area has established asymptotics for the number of walks in certain families of models by applying the techniques of analytic combinatorics in several variables (ACSV), where one encodes the generating function of a lattice path model as the diagonal of a multivariate rational function. Melczer and Mishna obtained asymptotics when the set of steps $S$ is symmetric over every axis; in this setting one can always apply the methods of ACSV to a multivariate rational function whose set of singularities is a smooth manifold (the simplest case). Here we go further, providing asymptotics for models with generating functions that must be encoded by multivariate rational functions with non-smooth singular sets. In the process, our analysis connects past work to deeper structural results in the theory of analytic combinatorics in several variables. One application is a closed form for asymptotics of models defined by step sets which are symmetric over all but one axis. As a special case, we apply our results when $d = 2$ to give a rigorous proof of asymptotics conjectured by Bostan and Kauers; asymptotics for walks returning to boundary axes and the origin are also given.

(Thursday 16:00)
An Axiomatic Unification of Graph Colouring

Tim E. Wilson*

*Monash University

Recent decades have seen an explosion in variations of graph colouring, including star, acyclic, frugal, parity-free, conflict-free, square-free, and anagram-free colouring. These types of colouring share many similarities yet, in most cases, are studied independently of each other. This talk presents a generalisation of graph colouring which can be used to prove results that can be directly applied to many types of colouring. The generalisation consists of five axioms which capture the notion of graph colouring as finite pattern avoidance.

Variants of graph colouring are formulated as sets of coloured graphs, called colour schemes, that satisfy the five axioms. The set of colour schemes has a lattice structure which encodes many of the relationships between types of graph colouring found in the literature. We explore the lattice of colour schemes and present novel variants of graph colouring that fill the gaps between existing variants. These new variants have previously unseen properties and answer some questions about the admissible behaviour of ‘reasonable’ types of graph colouring.

(Monday 11:00)

Graph and Poset Dimension Parameters

Alex Scott, David R. Wood*

*Monash University

The dimension of a poset $P$ is the minimum number of total orders whose intersection is $P$. We prove that the dimension of every poset whose comparability graph has maximum degree $\Delta$ is at most $\Delta \log^{1+o(1)} \Delta$. This result improves on a 30-year old bound of Füredi and Kahn [Order, 1986], and is within a $\log^{o(1)} \Delta$ factor of optimal. We prove this result via the notion of boxicity.

The boxicity of a graph $G$ is the minimum integer $d$ such that $G$ is the intersection graph of $d$-dimensional axis-aligned boxes. We prove that every graph with maximum degree $\Delta$ has boxicity at most $\Delta \log^{1+o(1)} \Delta$, which is also within a $\log^{o(1)} \Delta$ factor of optimal. We also show that the maximum boxicity of graphs with Euler genus $g$ is $\Theta(\sqrt{g \log g})$, which solves an open problem of Esperet and Joret [Graphs Combin. 2013] and is tight up to a constant factor.

The separation dimension of a graph $G$ is the minimum positive integer $d$ for which there is an embedding of $G$ into $\mathbb{R}^d$, such that every pair of disjoint edges are separated by some axis-parallel hyperplane. We prove a conjecture of Alon et al. [SIAM J. Discrete Math. 2015] by showing that every graph with maximum degree $\Delta$ has separation dimension less than $20\Delta$, which is best possible up to a constant factor. We also prove that graphs with separation dimension 3 have bounded average degree and bounded chromatic number, partially resolving open problems by Alon et al. [J. Graph Theory 2018].

(Tuesday 16:30)
The degree sequence of a random graph, and counting Latin rectangles

Kevin Leckey Anita Liebenau and Nick Wormald∗
∗Monash University

For random graphs, the distribution of their degree sequence was one of the first topics studied in depth. The distribution of degrees has close connections with the enumeration of graphs with given vertex degrees. Both problems are most often considered in the asymptotic sense, as the number of vertices goes to infinity. The asymptotic enumeration results have also found many uses in studying properties of random graphs, such as the structure of the giant connected component, and random matrix theory.

This talk will discuss a new approach to asymptotic enumeration by degree sequence that the speaker used with Anita Liebenau to confirm formulae that were conjectured in the early 1990’s. The new result establishes a simple model for the degree sequence of a random graph in terms of independent binomial variables, that was conjectured around the same time. Applying similar methods to enumeration of edge-coloured graphs, and adding Kevin Leckey as a coauthor, gave a formula for $k \times n$ Latin rectangles that improves the result announced by Godsil and McKay in 1984 and verifies one of their conjectures.

(Tuesday 10:30)

Stability of circulant graphs

Yan-Li Qin, Binzhou Xia∗ and Sanming Zhou
∗The University of Melbourne

A graph is said to be stable if its canonical double cover has no unexpected symmetries. Graph stability has been studied in the literature from different viewpoints. In this talk I will first review these viewpoints and then focus on the stability of circulant graphs. In particular, I will give an answer to a question of Wilson in 2008 on the stability of arc-transitive circulant graphs and infinitely many counterexamples to a conjecture of Marusic, Scapellato and Zagaglia Salvi in 1989.

(Monday 10:30)
Pentagonal subdivision turns any tiling on oriented surface into a tiling of pentagons. The construction appears in our classification of tilings of the sphere by congruent pentagons. It also gives pentagonal tilings of the sphere with evenly distributed vertices of degree $> 3$, typically with the property that the degrees of the vertices of every tile are $3, 3, 3, 3, d$, with $d > 3$. A natural question is the converse, i.e., whether any such tiling is constructed by pentagonal subdivision.

A more in depth study of the pentagonal subdivision leads to the more fundamental problem of certain pentagonal subdivision of quadrilateral tilings, which we call simple pentagonal subdivision. We find the necessary and sufficient conditions for the following questions:

1. When does a quadrilateral tiling admit a simple pentagonal subdivision, in terms of bipartite property.

2. When does a quadrilateral tiling admit a simple pentagonal subdivision, in terms of (the more calculable) degeneracy property of quadrilateral tiles.

3. When is a pentagonal tiling the result of simple pentagonal subdivision.

The last problem can then be used to answer our original question of pentagonal tilings with evenly distributed vertices of degree $> 3$.

(Tuesday 15:30)